Inverse discrete wavelet transform in image processing

A wavelet is a function that oscillates like a wave but is quickly attenuated. A wavelet is a function ψ of L2(R) who verifies the following admissibility condition:

$$\begin{array}{@{}rcl@{}} C_{\psi}=\int_{-\infty}^{+\infty} \frac{| \hat{\psi}(\xi)|^{2}}{\left| \xi\right| }d\xi < +\infty \end{array} $$

(2) Discrete wavelet transformThe discrete wavelet transform is based on the concept of multi-resolution analysis (MRA) introduced by Mallat [29]. The discrete wavelet transform (DWT) of image signals produces a non-redundant image representation, which provides better spatial and spectral localization of image formation, compared with other multi-scale representations such as Gaussian and Laplacian pyramid. Recently, the discrete wavelet transform has attracted more and more interest in image denoising. The DWT can be interpreted as signal decomposition in a set of independent, spatially oriented frequency channels. The signal S is passed through two complementary filters and produces two signals: approximation and details. This is called decomposition or analysis. The components can be assembled back into the original signal without loss of information. This process is called reconstruction or synthesis. The mathematical manipulation, which implies analysis and synthesis, is called a discrete wavelet transform and inverse discrete wavelet transform [30].1D discrete wavelet transformEvery analog signal x(t) with finite energy can be decomposed into a sum of shifted and dilated wavelet functions ψ(t) and shifted scale functions ϕ(t):

$$\begin{array}{@{}rcl@{}} x(t)=\sum_{k=-\infty}^{\infty}c(k)\phi(t-k)+\sum_{j=0}^{\infty}\sum_{k=-\infty}^{\infty}d(j,k)2^{\frac{j}{2}}\psi(2^{j}t-k) \end{array} $$

(3) where c(k) are scale coefficients and d(j,k) wavelet coefficients. This is a dyadic variant of the DWT. Scale and wavelet coefficients are calculated using scalar products:

$$\begin{array}{@{}rcl@{}} c(k)=\int_{-\infty}^{+\infty}x(t)\phi(t-k)dt \end{array} $$

(4)

$$\begin{array}{@{}rcl@{}} d(j,k)=2^{\frac{j}{2}}\int_{-\infty}^{+\infty}x(t)\psi(2^{j}t-k)dt \end{array} $$

(5) Hence, filter banks with perfect reconstruction property can be used as a simple realization of the DWT using low-pass and high-pass filters associated, respectively, to the scale function, and the wavelet function [5].2D discrete wavelet transformSeparable 2D discrete wavelet transform is the simplest form of the two-dimensional wavelet generalization. It consists of a standard 1D DWT applied to each row and then to each column as shown in Fig. 1. Fig. 1Block diagram of wavelet transformIn Fig. 1, if an image has N1 rows and N2 columns, decomposition results in four quarter-size images (N1/2×N2/2): details (LH, HL, HH) and approximation LL. Approximation LL is product of two low-pass filters and provides for an input to the next decomposition level. The reconstruction is performed in the opposite way: first on columns, then on rows. Separable 2D DWT has three wavelet functions (m and n are coordinates of the input image):

$$\begin{array}{@{}rcl@{}} \psi^{1}(m,n)=\phi(m)\psi(n) \hspace{0.25cm} LH wavelet, \end{array} $$

(6)

$$\begin{array}{@{}rcl@{}} \psi^{2}(m,n)=\psi(m)\phi(n) \hspace{0.25cm} HL wavelet, \end{array} $$

(7)

$$\begin{array}{@{}rcl@{}} \psi^{3}(m,n)=\psi(m)\psi(n) \hspace{0.25cm} HH wavelet, \end{array} $$

(8) and one scale function:

$$\begin{array}{@{}rcl@{}} \phi^{2}(m,n)=\phi(m)\phi(n) \end{array} $$

(9) associated to the approximation LL [31].An N level decomposition can be performed resulting in 3N+1 different frequency bands: LL is low frequency or approximation coefficients, and the wavelet image coefficients LH, HL, and HH which correspond, respectively, to vertical high frequencies (horizontal edges), horizontal high frequencies (vertical edges), and high frequencies in both directions (corners), as shown in Fig. 2. In Fig. 2, the number written next to the sub-band name shows the level. The next level of wavelet transform is applied to the low-frequency sub-band image LL only. For more details about the 2D discrete wavelet transform and 2D inverse discrete wavelet transform, the reader can refer to [5] and [32–34]. Fig. 2Sub-bands after two levels of wavelet decompositionOne of the main advantages of wavelets is that they allow complex information such as images to be decomposed into elementary forms at different positions and scales and subsequently reconstructed with high precision [5].The second main advantage of wavelets is that using fast wavelet transform based on filter banks [5], it is computationally efficient.Wavelet transform provides sparse representation for a large class of signals [5], and it is capable of revealing aspects of data that other signal analysis techniques miss the aspects like trends, breakdown points, and discontinuities in higher derivatives and self-similarity [35].Wavelets have the great advantage of being able to capture the energy of a signal in few energy transform values, it does not change the number of pixels required to represent the image and separate the information in a way that resembles the human visual system [36].Related workThe K-SVD denoising methodSparse and redundant representations model of signalGiven a signal y∈Rn and a over-completed dictionary D=[d1,d2,⋯,dk]∈Rn×k(n